On the use of permutation distances in metaheuristics

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1 Introduction

One of the challenging aspects in metaheuristics design is an adequate definition of solution *quality*, which has both cost and structural properties. Cost is always measured by an objective function, while structural properties are reflected by values of decision variables. From the application of successful metaheuristics, first and foremost scatter search (SCS, see [1]), it is well-known that structural properties play an import role when a *set* of solutions has to be evaluated during the course of the optimization. Those pooled solutions may be treated in a simultaneous way, e.g. in the so-called reference set of an SCS, or subsequently in *multi-(re)start* approaches, which means that new search trajectories are initialized upon solutions which have been proved to incorporate a higher quality and have been recognized as being elite during the search history. Algorithms which are evaluating solutions upon their distance to each other or to the incumbent best-known solution have proved to be very effective in the context of selecting or rejecting solutions (or their elements) (e.g. see [2, 3]).
In the current paper we want to concentrate on the quadratic assignment problem (QAP). The intended goal is a twofold one: firstly, the main focus lies on a methodic and statistical analysis, which evaluates the use of a permutation distance measure in the context of the QAP. Secondly, based on the preceding experimental findings, we try to design some useful heuristic components in order to improve a standard metaheuristic algorithm. The basis is the established robust tabu search code by Taillard [4].

2 Approach

Generally, we consider a given permutation problem and the following *basic multi-restart* algorithm (BMD) in which a very straightforward simple distance measure application is embedded.

Define x and y to be solutions of solution sets X and Y, respectively. Especially, X is the set of all solutions according to a problem-specific landscape. H denotes a heuristic procedure, $d(.)$ is the Hamming function and LB an input-parameter which can be varied according to percentages of a pre-calculated maximum distance LB_{max}:

Step 1	(Seeding) Generate $Y \subseteq X$.
Step 2	(Selection) Select an untreated x of Y.
Step 3	(Distance) Calculate the distance between x and $x*$, say $d(x, x*)$. If $d(x, x*) > LB$, i.e. some parameter lower-bound, then goto goto *Step 5*.
Step 4	(Solution) Apply H to x getting $x' := H(x)$. If x' improves on $x*$ (the currently best-known) then set $x* = x'$.
Step 5	(Iteration) If Y is not empty goto *Step 2* else stop and output x*.

Table 1: Algorithm BMD.

Assuming this simple experimental framework, several questions can be addressed. BMD also covers the total enumeration sub-algorithm. This is the case if the starting solution set Y comprises all solutions in the problem landscape, i.e. the general case $Y = X$, and the setting $LB := \infty$. As soon as $LB \leq LB_{max}$, rejections of starting solutions take place. The main questions addressed are: do useful strategies exist, which can be derived for the specific case with $Y \subset X$, where *strategies* means an efficient arrangement of X, and how should one adjust LB to achieve rejections which ensure an balanced outcome in terms of running time and solution quality.

3 Preliminary results

The first averaged observations were made for the traveling salesman problem (TSP) using the compound neighborhood greedy heuristic of [2], 50 small random instances and the total enumeration case $Y := X$. Note that this settings allows the calculation of *ex-post* probabilities of finding an optimum solution based on a specific rejection rate LB. Varying this rate in decades of percentages, revealed an interesting result: between the range of $0.55LB_{max}$ and $0.15LB_{max}$ this probability function is monotone increasing, which gives rise to the assumption that there might be interesting regions of LB in the specific case too, i.e. where (naturally) a setting $Y := X$ has to be avoided.
Currently, we investigate the adaptation of this TSP approach to the QAP, above all, ways to statistically analyze the outcome of necessarily limiting the seeding (and selection) process to $Y \subset X$ when treating real benchmark QAP instances from the OR-library. At the first sight, it appears that no analog optimization behavior can be expected underlying a reasonable significance. It is expected that this is a direct consequence of the composition technique used which defines Y.

References

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