A METHODOLOGY FOR DESIGNING REAL-LIFE APPLICATIONS

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CONTENT OF THE TALK

Problem definition

Data collection

Problem modelling

Neighbourhoods

Key element in metaheuristics ; basic, reduced, expanded

Large problem instances

Problem decomposition ; POPMUSIC

Multiobjective optimization

Path relinking

Dynamic problems

Adaptive memory programming

Sythesis

UNDERSTAND THE PROBLEM

Most difficult part of a practical project

The answer is not necessarily in relation with the original question

— What type of lorries do I have to buy ?

— Sell the two oldest of your fleet !

Visit the operational unit

An efficient way to know what is already done

Identify the true constraints

Example: (almost) no carrying company respects the laws

Identify the right objective(s)

The customer typically want to diminish the fixed, incompressible costs

Collect data

Ask for the right data

Concise

Coherent

Typical mistake : asking for a complete distance matrix

Too many data

Data not coherent (e.g. triangle inequalities not respected)

Typically takes more than 50% of the time of the project

USE A GOOD MODEL

Turbine runner balancing :

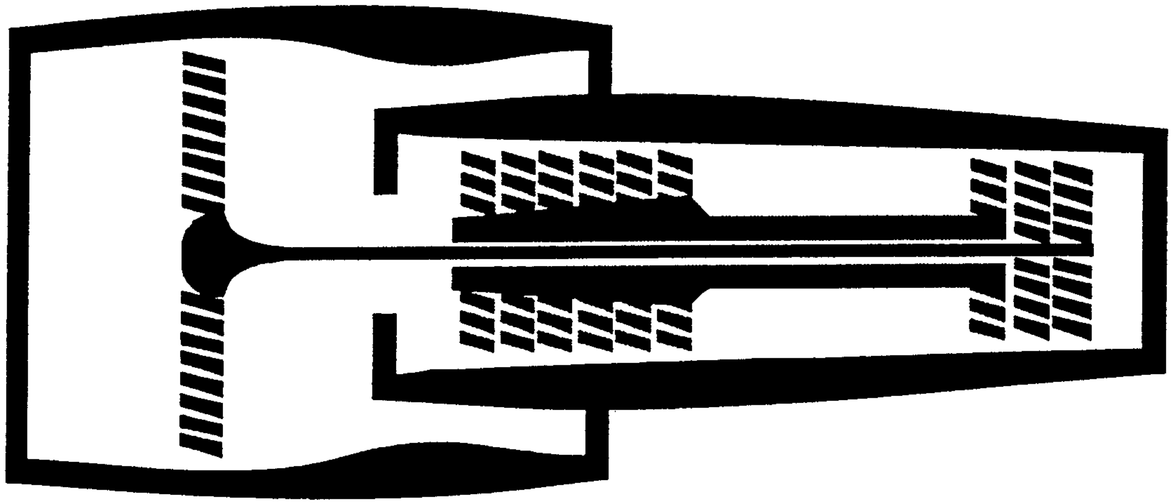
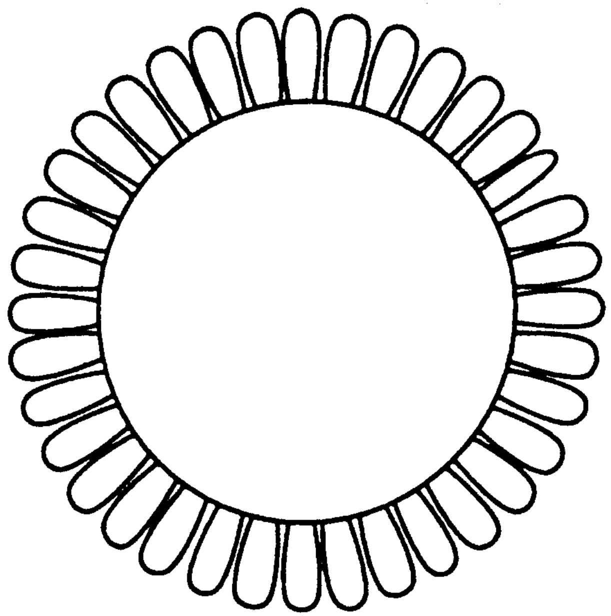


Fig. 1. Schematic figure of a jet engine.



Source : Mason & Rönnqvist, C&OR 24, 1997

n blades of weight w_i ($i = 1, \ldots, n$)

n angular positions $\theta_i = i/2\pi$ ($i = 1, \ldots, n$) or, more generally : cartesian coordinates (x_i, y_i)

Objective : find a positions p_i ($i = 1, \ldots, n$) for each blade that minimizes : $\left(\sum_{i=1}^{n} w_i \cdot x_{p_i}\right)^2 + \left(\sum_{i=1}^{n} w_i \cdot y_{p_i}\right)^2$

TURBINE BALANCING

Alternate formulation (Laporte & Mercure, EJOR 35, 1988) :

Quadratic assignment problem with :

flow matrix $f_{ij} = w_i w_j$

distance matrix $d_{ij} = cos(\theta_i - \theta_j)$

Objective : find a permutation p that minimizes : $\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} \cdot d_{p_i p_j}$

Less general

Works only for angular positions

Container vessel loading ?

More complex

Objective computation $O(n^2)$ versus initial formulation $O(n)$

DIFFERENCE BETWEEN ACADEMIC AND REAL-LIFE

Academic QAP

Data : Flow matrix (f_{ij}) ; distance matrix (d_{ij}) ; $n = 25 \ldots 150 \ldots (729)$

Constraints : $p \in$ permutation

Objective : Minimize $\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} \cdot d_{p_i p_j}$

Real-life QAP

Data : Flow matrices : $(f_{ij}^1), (f_{ij}^2)$; distances matrices : $(d_{ij}^1), (d_{ij}^2)$;

$n = 60, \ldots, 200, \ldots, (32000)$

Constraints : Blocks $p = \left(p_{b_1^1} \cdots p_{b_{n_1}^1}, p_{b_1^2} \cdots p_{b_{n_2}^2}, \ldots, p_{b_1^m} \cdots p_{b_{n_m}^m}\right)$

Computational time : 3' at most

Objectives : Minimize $\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}^1 \cdot d_{p_i p_j}^1$ and $\sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij}^2 \cdot d_{p_i p_j}^2$

Modelling : Hard and Soft Constraints

Find the right balance :

Too many hard constraints

Difficult to find a feasible solution

Difficult to move from one solution to another

Too many soft constraints

Solutions not feasible in real life

Finding good penalty associated with a violated constraint

Balance depends on solving method :

Constraint programming

As many hard constraints as possible (reduce solutions space)

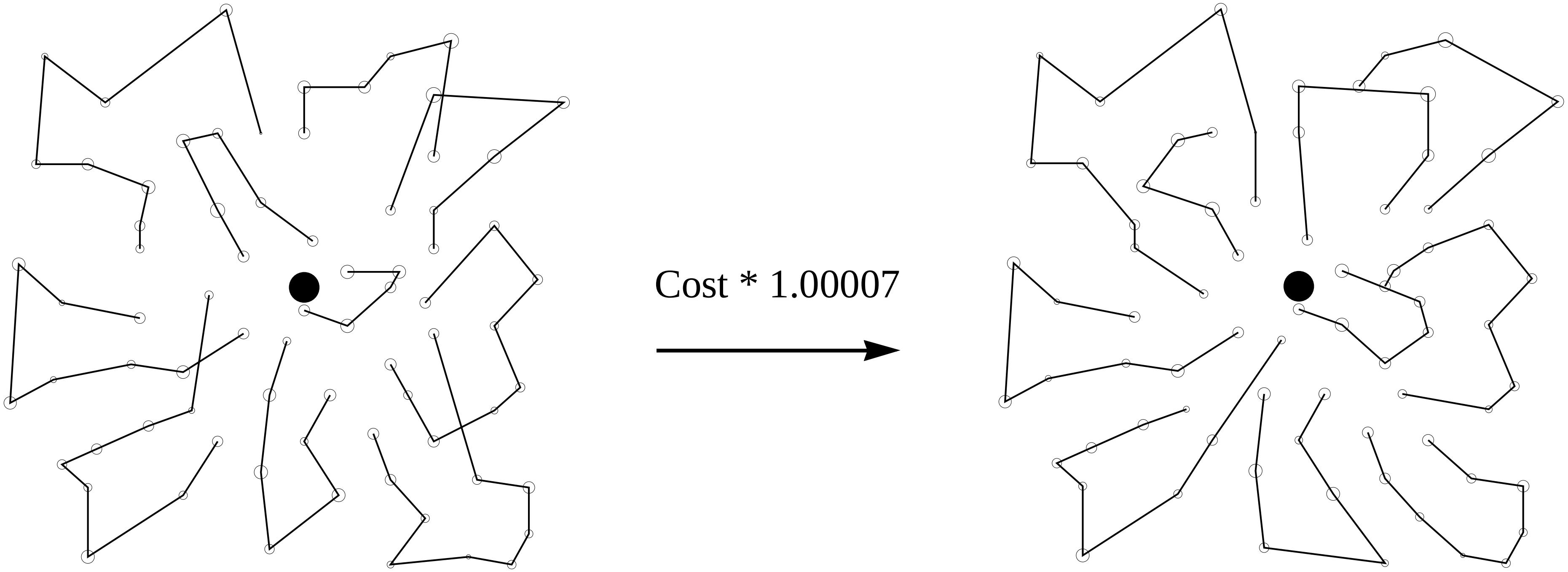
Noising methods

Many soft constraints (for adding noise in the objective)

NEIGHBOURHOOD

Definition of good neighbourhoods is a key element in many practical applications

A local optimum (relatively to an appropriate neighbourhood) is often convenient in real-life applications (and there is often no time to do much more)



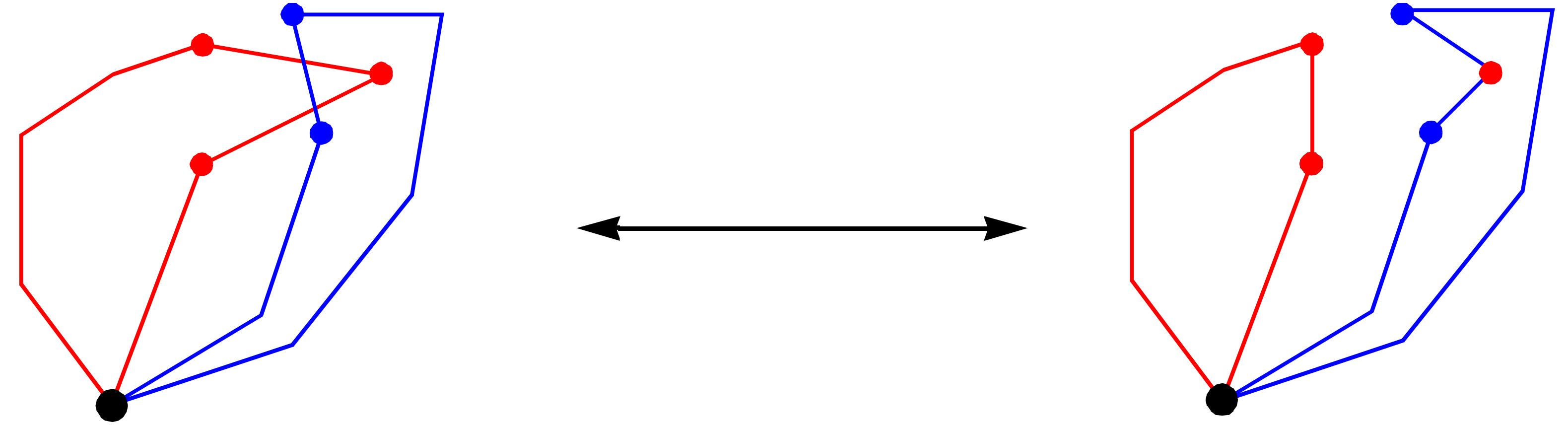
Interesting academic property : this instance is hard to solve exactly

Real-life interest : ?

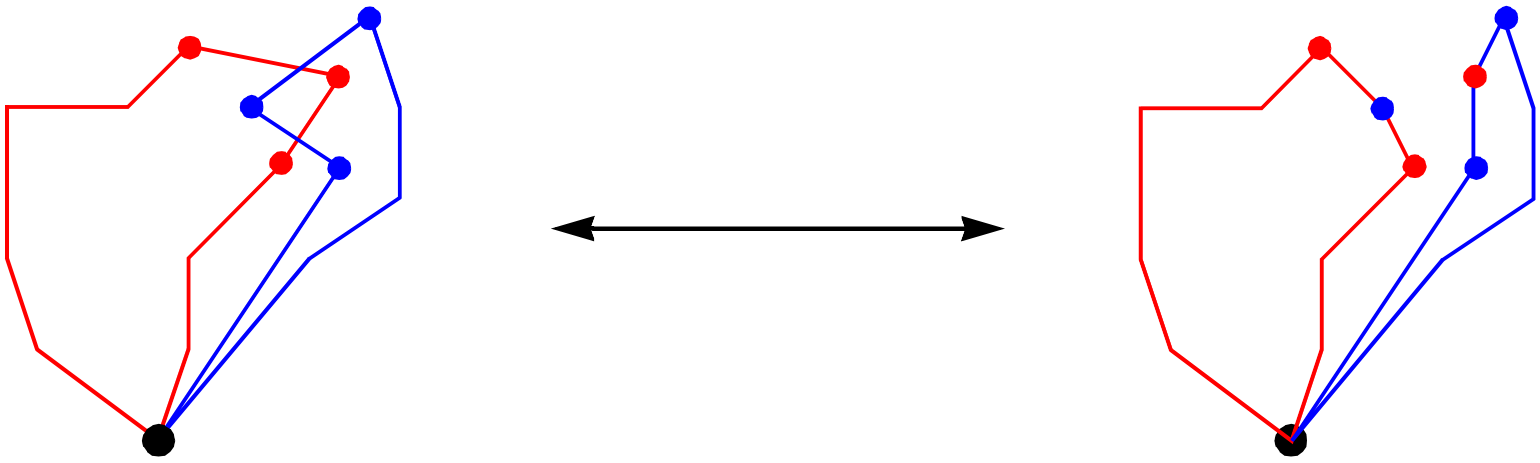
Simple Neighbourhoods

Examples for the VRP (*n* customers, *m* tours)

Insertion (1-interchange) : $O(nm)$ neighbours



Exchange (2-interchange) : $O(n^2)$ neighbours

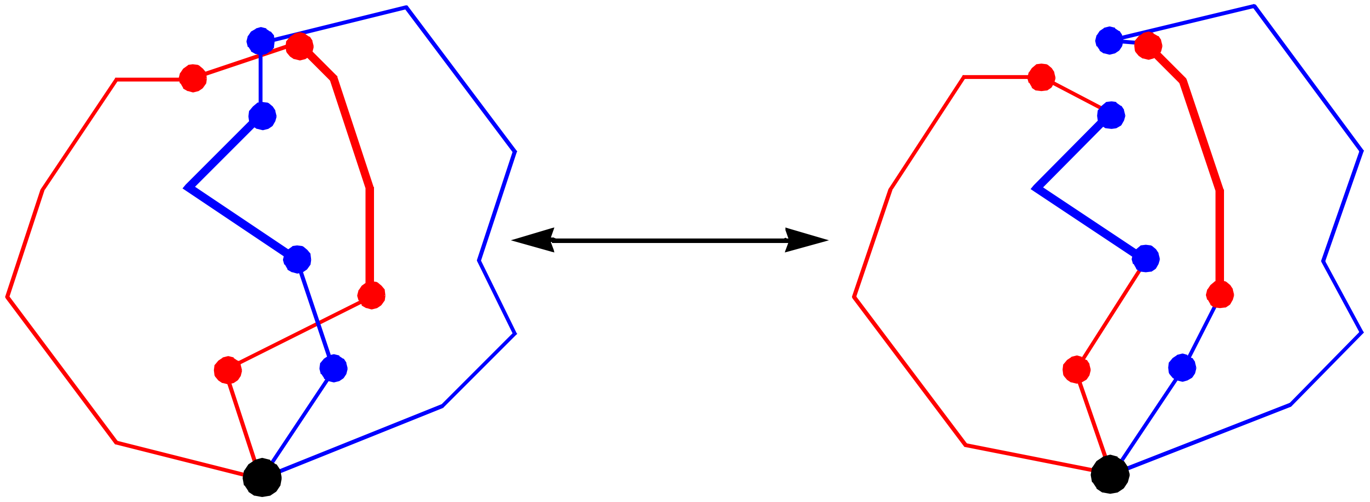


Generalization : λ-interchange : $O(n^\lambda)$ neighbours.

RESTRICTION OF NEIGHBOURHOOD SIZE

Candidate list

Example 1 : CROSS-neighbourhood : $O(n^4/m^2)$ neighbours



Example 2 : Granular tabu search (Toth & Vigo 1998)

Consider only the shorter edges adjacent to each customers

$O(n^2) \rightarrow O(n)$

NEIGHBOURHOOD EXPANSION : COMPOSITE MOVES

Ejection Chains

Avoid atomic changes

Expand neighbourhood size without increasing complexity too much

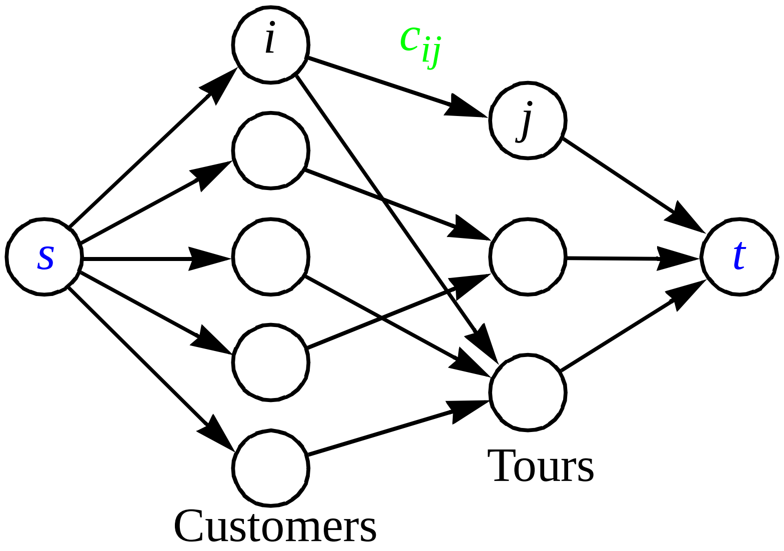
Perform jumps in solution space

Example for the VRP :

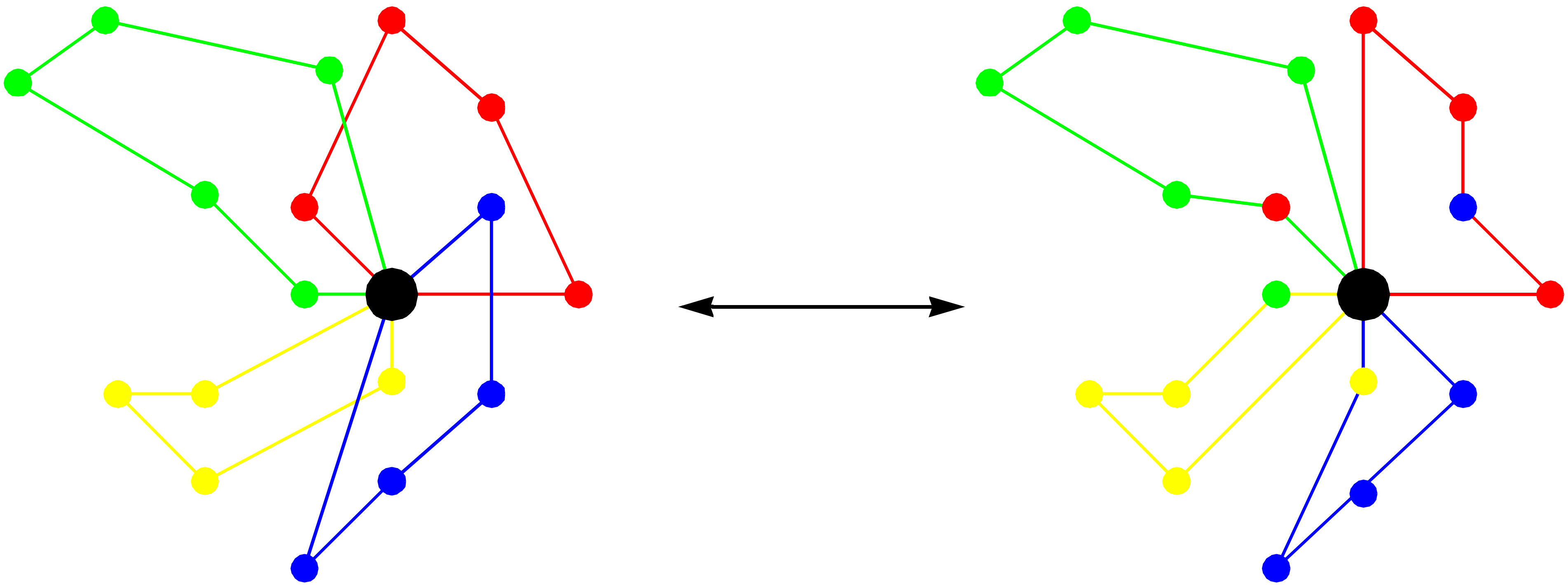
Network flow model (Xu & Kelly, Transp. Sci 30, 1996)

Find the max flow from s to t with lowest cost

c_{ij} : cost of removing customer i and inserting it in tour j



Large Neighbourhood : “ Rotation ”



Finding the best rotation with ejection chains : $O(nm^2)$

DECOMPOSITION OF LARGE PROBLEMS

Large neighbourhood, POPMUSIC

Solution composed of *parts* $s_1, \ldots, s_p$

Take a *seed part* s_i

Build a sub-problem : s_i + r “closest” parts

Optimize sub-problem

Recompose solution

Repeat with another seed part

Taillard 1993, Shaw 1998

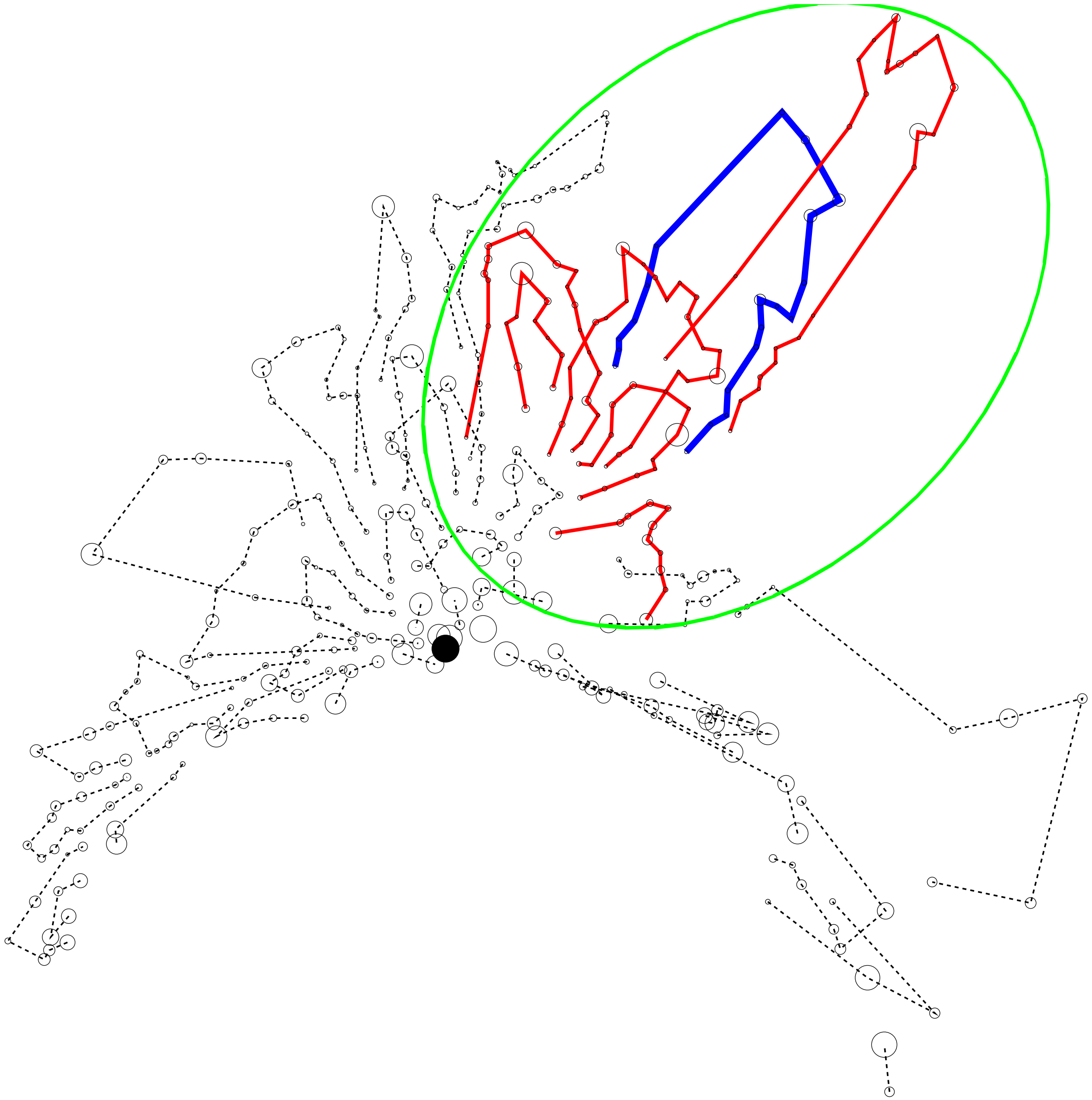
VRP

Taillard & Voss 2001

POPMUSIC

Taillard, Taillard & Wälti 2003

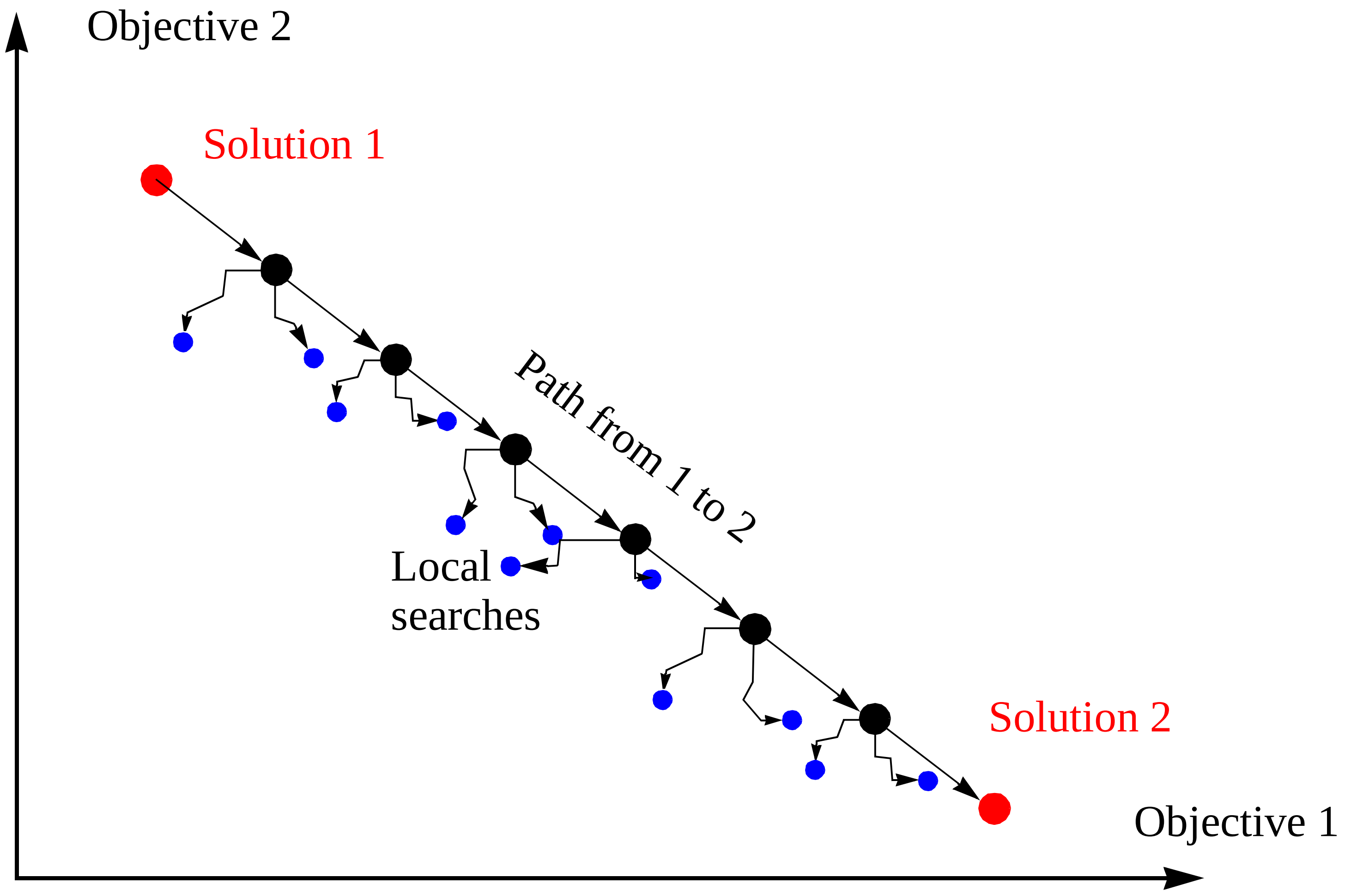
Location-allocation



FROM SINGLE TO MULTI-OBJECTIVE OPTIMIZATION

Path relinking

X. Gandibleux, H. Morita, N. Katoh, MIC 2003



DYNAMIC PROBLEMS

Often, practical problems are dynamic (e.g. arrival of new customers, job finished, breakdown)

Adaptive Memory Programming (AMP)

General algorithm

Initialize memory

Repeat, until a stopping criterion met :

Build a provisory solution with the help of informations in memory

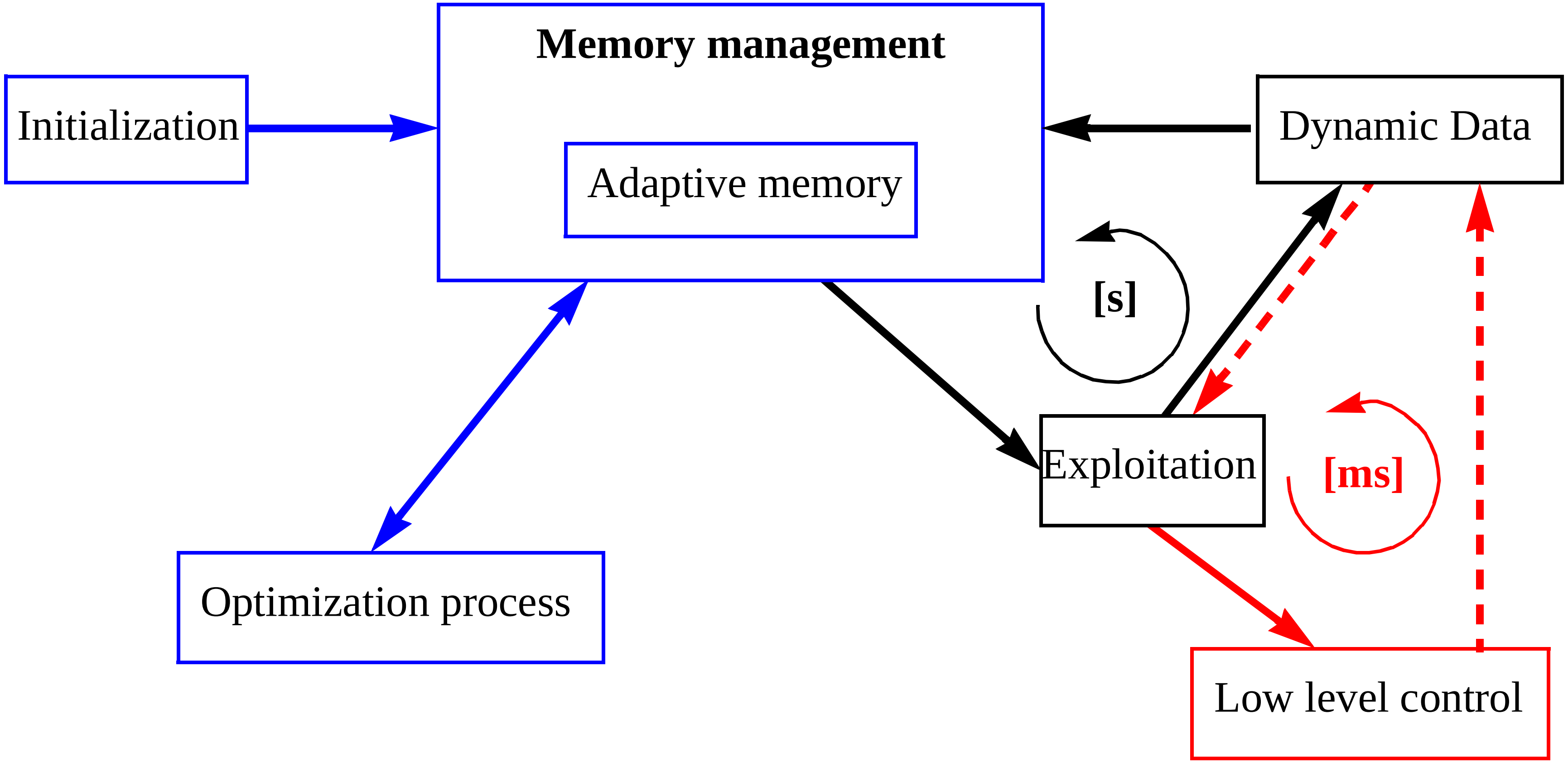
Improve provisory solution with a local search

Update memory with informations obtained with new solution

Ant systems	Memory ≡ pheromone traces
Evolutionary algorithms, Scatter Search	Memory ≡ population of solutions
Vocabulary building	Memory ≡ pieces of solutions

Provocative comment : Memories and building procedures for all these methods are equivalent.

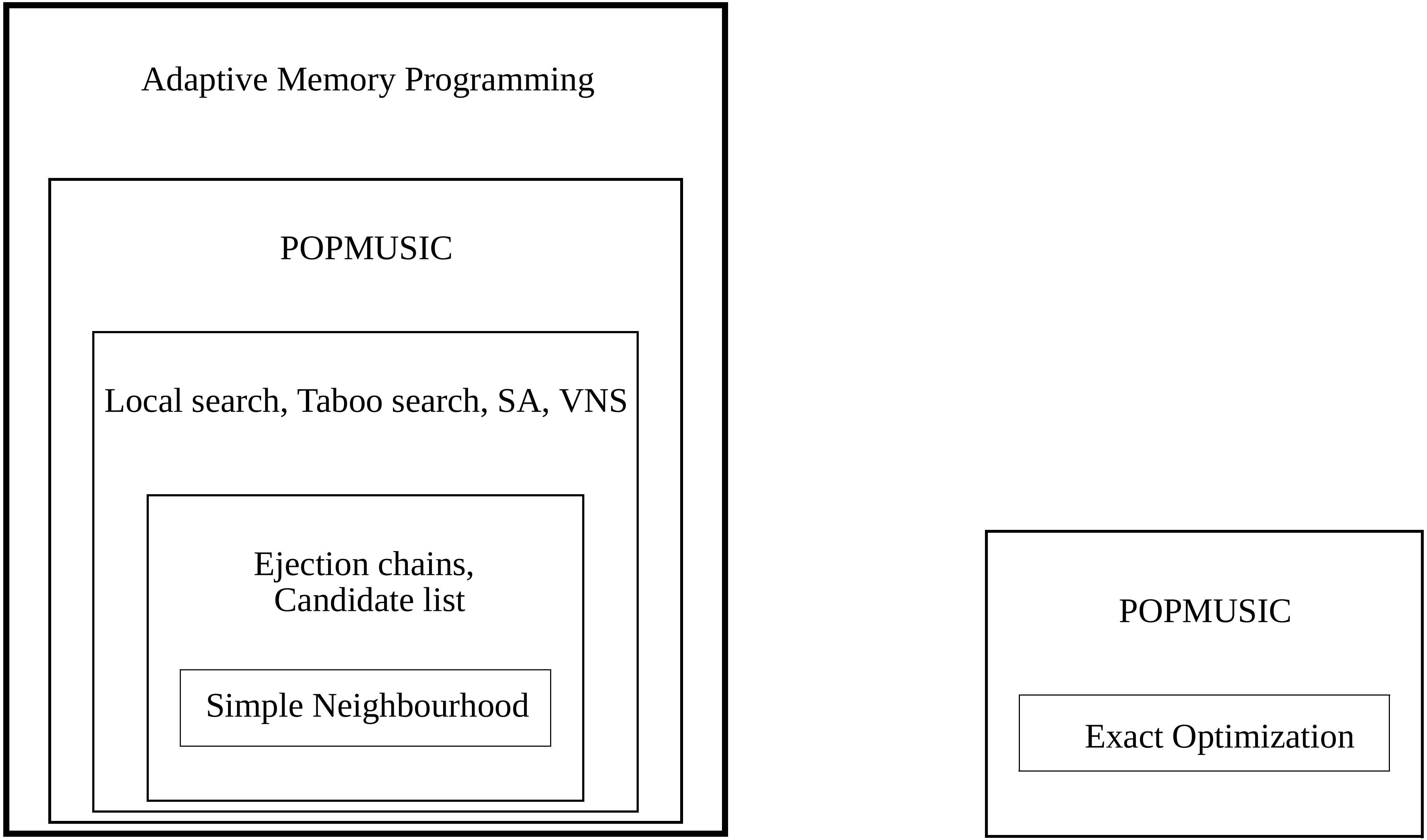
AMP TECHNOLOGY



Computing power not used by real time

Real time management

CONCLUSION : METHODOLOGY PROPOSAL



Design of a complex method

A less complex design