Appendix. Mathematical formulations

A. Group positioning problem

This Section provides mathematical formulations for the group positioning problem.

A.1. Sets

- I: Logical vehicles to be placed.
- S: Vehicle series (including 0 indicating that a lane is empty).
- V: Lanes where the vehicles can be parked.
- $V_s \subset V, s \in S$: Subset of lanes where vehicle series s can be parked.

A.2. Data

- $s(i)$: Series of vehicle i.
- l_i: Length of vehicle i.
- c_v: Capacity (length) of lane v.

A.3. Variables

- x_{iv}: Vehicle $i \in I$ is on lane $v \in V_{s(i)}$ $(x_{iv} = 1)$ or not $(x_{iv} = 0)$
- y_{sv}: Lane $v \in V_s$ is occupied by vehicle series s $(y_{sv} = 1)$ or not $(y_{sv} = 0)$

For having a linear objective, it is possible to introduce variables $z_{ss'v}$ whose value must be set to: $z_{ss'v} = y_{sv} \cdot y_{s'(v+1)}$. This is automatically done by *Gurobi* and does not significantly influence its computational time.

A.4. Constraints

- A vehicle is assigned to exactly one lane:

$$\sum_{v \in V_{s(i)}} x_{iv} = 1 \quad \forall i \in I \tag{1}$$

- A lane is occupied by exactly one series:

$$\sum_{v} y_{sv} = 1 \quad \forall s \in S \tag{2}$$

- A vehicle can be placed only on a lane with the right series:

$$x_{iv} \leq y_{sv} \quad \forall i \in I, \forall v \in V_{s(i)}, \forall s \in S \tag{3}$$

- The sum of vehicle lengths on a lane cannot exceed the lane capacity:

$$\sum_{i} l_i \; x_{iv} \leq c_v \quad \forall v \in V_{s(i)} \tag{4}$$

A.5. Objectives

- First objective consists of grouping vehicle series, i.e. minimizing the number of different series in adjacent lanes:

$$obj_1: \text{ Minimize } \sum_{s\neq 0}\sum_{s'\neq s}\sum_{v} y_{sv}\cdot y_{s'(v+1)} \tag{5}$$

- Second objective is to minimize the number of occupied lanes (or maximizing free lanes):

$$obj_2: \text{ Minimize } \sum_{s\neq 0}\sum_{v} y_{sv} \tag{6}$$

- Third objective is to minimize the remaining space on each lane (or maximizing the occupied space):

$$obj_3: \text{ Minimize } \sum_{s\neq 0}\sum_{v} c_v\ y_{sv} \tag{7}$$

- The global objective for the group positioning is:

$$\text{Minimize } \quad p_1\ obj_1 + p_2\ obj_2 + obj_3, \text{where } p_1 \text{ and } p_2 \text{ are weights such that } p_1 \gg obj_2 \text{ and } p_2 \gg obj_3. \tag{8}$$

B. Schedule assignment problem

This Section provides a mathematical formulation for the schedules assignment problem.

B.1. Sets

- I: Schedules (or physical vehicles) to be placed.
- V: Lanes where the vehicles must be parked.
- $V_i \subset V, i \in I$: Subsets of lanes where vehicle i can be parked.
- $N \subset V$: Lanes for which the vehicle in front must leave before those in front of the next adjacent lane.
- $P \subset V$: Lanes where the vehicle in front must leave before those in front of the previous adjacent lane.

B.2. Data and parameters

- h_i: Departure time of vehicle i ($i \in I$).
- h_{min}: Minimal difference of time between departures on the same lane ($h_{min} = 10$).
- h_{ideal}: Ideal difference of time ($h_{ideal} = 20$).
- n_v: Number of vehicles to be placed on lane v ($v \in V$)
- t_{ij}: Schedule type of vehicle i and j are similar ($t_{ij} = 1$) or not ($t_{ij} = 0$).
- w_{ij}: Reward for respecting h_{min}

$$w_{ij} = \begin{cases} 1.5\cdot h_{min} & h_{min} \leq h_j - h_i \leq h_{ideal} \\ h_{min} & h_j - h_i > h_{ideal} \\ -4\cdot(h_{min} - (h_j - h_i)) & h_j - h_i < h_{min} \end{cases}$$

B.3. Variables

- $x_{ivp} = \begin{cases} 1 & \text{if schedule } i \text{ is on lane } v \text{ at position } p \\ 0 & \text{otherwise} \end{cases} \quad (i \in I, v \in V_i, 1 \leq p \leq n_v)$

B.4. Constraints

- A vehicle is assigned to exactly one position:

$$\sum_{i|v\in V_i} x_{ivp} = 1 \quad \forall v \in V \ \forall p \tag{9}$$

- A position is occupied by exactly one vehicle:

$$\sum_{p} x_{ivp} = 1 \quad \forall i \in I, \forall v \in V_i \tag{10}$$

- The departure time of any vehicle must be prior to the vehicle following it:

$$h_i \ x_{ivp} \leq \sum_{j\neq i} h_j \ x_{jv(p+1)} \quad \forall i \in I, \forall v \in V_i, 1 \leq p < n_v \tag{11}$$

- Weak blocking constraints:

$$h_i \ x_{iv1} \leq \sum_{j\neq i} h_j \ x_{j(v+1)1} \quad \forall v \in N, \forall i \in I \tag{12}$$

$$h_i \ x_{iv1} \leq \sum_{j\neq i} h_j \ x_{j(v-1)1} \quad \forall v \in P, \forall i \in I \tag{13}$$

B.5. Objectives

- Maximize the number of schedules of a same type (color) on each lane.

$$obj_1: \text{ Maximize} \quad \sum_{i} \sum_{j\neq i} \sum_{v\in V_i} \sum_{p=1}^{n_v-1} t_{ij} \ (x_{ivp} \cdot x_{jvp_{+1}}) \tag{14}$$

- Maximize the number of schedules of a same type between successive lanes (i.e. comparing the last vehicle of a lane with the first vehicle of the next lane):

$$obj_2: \text{ Maximize} \quad \sum_{i\in I} \sum_{j\neq i} \sum_{v,v+1\in V_i} t_{ij} \ (x_{ivn_v} \cdot x_{j(v+1)1}) \tag{15}$$

- Maximize best practices between any two departures (penalizing departures that do not respect a minimal time and rewarding departures that respect an ideal time):

$$obj_3: \text{ Maximize} \quad \sum_{i} \sum_{j\neq i} \sum_{v\in V_i} \sum_{p=1}^{n_v-1} w_{ij} \ (x_{ivp} \cdot x_{jv(p+1)}) \tag{16}$$

- Global objective:

$$\text{Maximize} \quad m \ (obj_1 + obj_2) + obj_3, \text{ where } m \text{ is such that } m \gg obj_3 \tag{17}$$